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Influence of Non-ideal Blackbody Radiator Emissivity and a Method for its Correction

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Abstract Demands for accurate temperature measurement and calibration are increasing along with the wider use of radiation thermometry in industry. However, the deviation of a 'blackbody' radiator emissivity from the emissivity of an ideal blackbody remains one of the main uncertainty contributions in the calibration of radiation thermometers, although the performance of blackbody radiators has been continually improving. Nevertheless, the influence of this deviation was often ignored due to the complexity of the correction. In this paper, general methods to evaluate the influence of the emissivity deviation of a blackbody radiator from unity for typical radiation thermometer models are described. An approximate practical method for wide-band radiation thermometers is proposed. Moreover, the concept of equivalent wavelength and the corresponding calculation method are introduced to simplify the mathematical model. The calculation result and a mathematical expression for the equivalent wavelength applicable to most popular radiation thermometers with a spectral range of 8–14 μm are given. The analysis and calculation show that the influence of blackbody radiator emissivity on longer working-wavelength radiation thermometer calibrations at mid or high temperatures cannot be ignored.

Keywords Blackbody radiator · Correction · Effective wavelength · Emissivity · Equivalent wavelength · Radiation thermometer · Uncertainty

1 Introduction

Demands for accurate temperature measurement and calibration are increasing more and more along with the widespread use of radiation thermometry in industry.

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Blackbody radiators (BRs) play an important role in radiation thermometry and radiometry. The deviation of the blackbody radiator emissivity (BRE) from the emissivity of an ideal blackbody remains one of the main uncertainty sources [1,2] influencing the measurements and the calibrations of radiation thermometers (RTs), although the performance of BRs has been continually improving.

Nevertheless, the influence of non-ideal emissivity is sometimes ignored, especially in the calibration of wide-band radiation thermometers (WBRT), due to the complexity of the correction, which often leads to an error that, unfortunately, is not negligible. However, the necessary spectral responsivity information may only be given in a specification sheet as the nominal spectral range of a WBRT or the working wavelength of a narrow-band RT, is costly and complicated to measure for most industrial users of radiation thermometers, and is impractical for thermometers that read directly the temperature. Simple methods to characterize the spectral responsivity were presented by Saunders [3]. With these methods, the radiation thermometer function can be adequately expressed by two spectral responsivity parameters, the mean wavelength and variance, and these two parameters for the low-temperature direct-reading thermometer with adjustable emissivity can be determined.

In this paper, a general method to evaluate and correct the influences of the deviation of the BRE from unity on measurement and calibration is described. The correction is usually complicated in the classical model for a WBRT; therefore, an approximate practical model is proposed to make it feasible. The concept of equivalent wavelength, which is different from that of effective wavelength but can be used in more complex cases, was introduced to simplify the mathematical model and calculation procedure for WBRTs. The calculation results and the mathematical expressions for the equivalent wavelength of the most popular WBRTs with a spectral range of 8 to 14 μm are given. These have been applied to the WBRT calibration at the National Institute of Metrology (NIM), China.

2 Calibration of Radiation Thermometers

The blackbody radiation laws provide the basis for radiation thermometry. There are two calibration methods for RTs: (a) measurements of BRs whose true temperature is known and (b) measurements of BRs whose radiance temperature is determined by a reference RT. In method (a), the BRs can be fixed-point BRs or variable-temperature BRs for which a platinum resistance thermometer, thermocouple, or other contact thermometer is employed as the reference thermometer. In method (b), the radiance temperature measured by the reference RT is different from the true BR temperature, as is the radiance temperature measured by the RT under calibration for both methods (a) and (b). The radiance temperature depends on the BRE deviation from unity and on the spectral responsivity of the RT. In method (a), corrections must be applied to the BR temperature in order to compare it with the radiance temperature measured by the RT under calibration, while in method (b), if both RTs have the same or similar spectral responsivity, the radiance temperatures can be compared directly, otherwise individual corrections must be applied.

Method (a) with variable temperature BRs is widely used and contact thermometers are employed to measure the BR temperature in most cases. However, the reading of a calibrated RT is different from the true BR temperature during the calibration, even when using an accurate RT. The influence of BRE deviation from unity is an important uncertainty factor for calibration and is sometimes dominant for precision calibrations.

3 Influence of BRE and Its Correction

The non-ideal BRE affects the calibration of RTs in two ways. First, the radiation from a BR will be less than that from an ideal blackbody radiator. Second, the reflection of ambient radiation from the BR adds to the radiation emitted by the BR. Both these effects need to be treated together because for an isothermal blackbody cavity they are completely correlated, and for a non-isothermal cavity, they are highly correlated.

To simplify the model used to analyze the influence of BRE on calibration, the following conditions were assumed: an isothermal blackbody cavity, a given temperature and emissivity for the blackbody cavity, and a known ambient temperature.

Three typical distributions of the spectral responsivities of the RTs were considered: (a) monochromatic, (b) neutral (total radiation type), and (c) bandpass. Multiband RTs are not considered in this paper. When calibrating, both the emitted and the reflected radiations from a BR are measured by a RT. According to Planck's law and the Stefan-Boltzmann law, the relationships between the radiance temperature (the indicated temperature for an accurate RT), T_{rad}, for the three spectral responsivity distributions mentioned above and the true BR temperature, T_{bb}, are, respectively, the following:

$$L_{\mathrm{b}}(\lambda, T_{\mathrm{rad}}) = \varepsilon L_{\mathrm{b}}(\lambda, T_{\mathrm{bb}}) + (1-\varepsilon) L_{\mathrm{b}}(\lambda, T_{\mathrm{am}}) \tag{1}$$

$$T_{\mathrm{rad}}^4 = \varepsilon T_{\mathrm{bb}}^4 + (1-\varepsilon) T_{\mathrm{am}}^4 \tag{2}$$

$$\begin{aligned}\int_0^\infty L_b(\lambda, T_{\mathrm{rad}}) R(\lambda) \mathrm{d}\lambda &= \int_0^\infty \varepsilon L_{\mathrm{b}}(\lambda, T_{\mathrm{bb}}) R(\lambda) \mathrm{d}\lambda \\ &\quad + \int_0^\infty (1-\varepsilon) L_{\mathrm{b}}(\lambda, T_{\mathrm{am}}) R(\lambda) \mathrm{d}\lambda \end{aligned} \tag{3}$$

$L_b(\lambda, T)$ is the spectral radiance of a blackbody at temperature T, λ is the wavelength in vacuum, ε is the BRE, T_{am} is the ambient temperature, and $R(\lambda)$ is the relative spectral responsivity of the RT. For the RTs with monochromatic or neutral spectral responsivity, the radiance temperature of the BR can be expressed as a direct analytical function of ε, T_{bb}, and T_{am},

$$T_{\mathrm{rad}} = F(\varepsilon, T_{\mathrm{bb}}, T_{\mathrm{am}}) \tag{4}$$

For a WBRT, the relationship also depends on the variable $R(\lambda)$,

$$T_{\text{rad}} = F(\varepsilon, T_{\text{bb}}, T_{\text{am}}, R(\lambda)) \tag{5}$$

In general, this cannot be expressed in a simple analytical form, but a method for its calculation is described later.

The difference ΔT between the radiance temperature and the true temperature of a BR caused by the BRE and the RT's spectral responsivity is

$$\Delta T = T_{\text{rad}} - T_{\text{bb}} \tag{6}$$

where the reading of the RT, T_{rad}, indicates the radiance temperature of a measured BR for a monochromatic or bandpass RT or the radiation temperature of a measured BR for a total radiation RT.

4 Influence of BRE for WBRT

4.1 Integral Calculation Method

To calculate the radiance temperature, T_{rad}, of a WBRT according to Eq. 3, the spectral responsivity of a WBRT, $R(\lambda)$, should be known, although it is costly and complicated to measure.

An approximate practical method is proposed to calculate the reading of a WBRT during its calibration to make the calculation feasible. Assuming that the spectral responsivity of a WBRT can be represented by a rectangular distribution, Eq. 3 can be simplified to

$$\int_{\lambda_1}^{\lambda_2} L_{\text{b}}(\lambda, T_{\text{rad}})\text{d}\lambda = \int_{\lambda_1}^{\lambda_2} \varepsilon L_{\text{b}}(\lambda, T_{\text{bb}})\text{d}\lambda + \int_{\lambda_1}^{\lambda_2} (1-\varepsilon) L_{\text{b}}(\lambda, T_{\text{am}})\text{d}\lambda \tag{7}$$

where λ_1 and λ_2 are the lower and upper limiting wavelengths of the rectangular spectral responsivity, respectively. According to Eq. 7, T_{rad} can be solved by an iterative process using numerical integration. Thus, the deviation ΔT caused by non-ideal BRE can be calculated by Eq. 6.

WBRTs with an 8 to 14 μm spectral range are the most popular WBRTs. Table 1 shows the calculated deviation ΔT of a WBRT of 8 to 14 μm rectangular spectral responsivity for a BRE of 0.99 and an ambient temperature of 20°C. For a BRE other than 0.99, the deviation ΔT is given by

$$\Delta T(\varepsilon) \approx \Delta T(0.99)\frac{1-\varepsilon}{1-0.99} \tag{8}$$

Table 1 Calculation of the deviation ΔT of an 8 to 14 μm rectangular spectral responsivity WBRT with a BRE of 0.99 and an ambient temperature of 20°C

Blackbody radiator temperature, t_{bb}(°C)	Deviation, Δt(°C)	Blackbody radiator temperature, t_{bb}(°C)	Deviation, Δt(°C)
−50	1.20	300	−1.92
−25	0.60	400	−2.61
0	0.22	500	−3.35
20	0.00	600	−4.12
60	−0.34	800	−5.74
100	−0.63	1, 000	−7.44
200	−1.27	1, 200	−9.21

4.2 Equivalent Wavelength Method

For frequent or real-time calculation, Eq. 7 and Table 1 are not convenient for use. An intermediate value was introduced, based on the integral intermediate value theorem (integral mean-value theorem) [4], which we will refer to as the equivalent wavelength, λ_e, in this paper. $\lambda_e \in (\lambda_1, \lambda_2)$. The equivalent wavelength is defined by

$$\begin{aligned}&\int_{\lambda_1}^{\lambda_2} [L_b(\lambda, T_{rad}) - \varepsilon L_b(\lambda, T_{bb}) - (1-\varepsilon) L_b(\lambda, T_{am})]\, d\lambda \\ &= [L_b(\lambda_e, T_{rad}) - \varepsilon L_b(\lambda_e, T_{bb}) - (1-\varepsilon) L_b(\lambda_e, T_{am})]\,(\lambda_2 - \lambda_1)\end{aligned} \tag{9}$$

Combining this expression with Eq. 7, an equation whose mathematical form is the same as that of Eq. 1 can be deduced,

$$L_b(\lambda_e, T_{rad}) = \varepsilon L_b(\lambda_e, T_{bb}) + (1-\varepsilon) L_b(\lambda_e, T_{am}) \tag{10}$$

The equivalent wavelength λ_e can be calculated from Eq. 7 by numerical integration using an iterative process. The results of calculating the equivalent wavelength for an 8 to 14 μm rectangular spectral responsivity WBRT for a BRE equal to 0.99 and an ambient temperature of 20°C are shown in Table 2. Using the equivalent wavelength, Eq. 10 can replace Eq. 7 when calculating the deviation ΔT.

Fitting the data in Table 2, the equivalent wavelength can be expressed by

$$\lambda_e \approx \sum_{i=0}^{6} a_i t_{bb}^i \tag{11}$$

where $a_0 = +10.418\,\mu$m, $a_1 = -3.0945 \times 10^{-3}$ $\mu\text{m} \cdot {}^\circ\text{C}^{-1}$, $a_2 = +9.2370 \times 10^{-6}$ $\mu\text{m} \cdot {}^\circ\text{C}^{-2}$, $a_3 = -18.313 \times 10^{-9}$ $\mu\text{m} \cdot {}^\circ\text{C}^{-3}$, $a_4 = +21.606 \times 10^{-12}$ $\mu\text{m}\cdot{}^\circ\text{C}^{-4}$, $a_5 = -13.415 \times 10^{-15}$ $\mu\text{m}\cdot{}^\circ\text{C}^{-5}$, and $a_6 = +3.3437 \times 10^{-18}$ $\mu\text{m}\cdot{}^\circ\text{C}^{-6}$. Simplified further, the equivalent wavelength can be approximated as

$$\lambda_e \approx 10\,\mu\text{m} \tag{12}$$

Table 2 Calculation of λ_e for an 8 to 14 μm rectangular spectral responsivity WBRT at an ambient temperature of 20°C

Blackbody radiator temperature, t_{bb}(°C)	Equivalent wavelength, λ_e (μm)	Blackbody radiator temperature, t_{bb}(°C)	Equivalent wavelength, λ_e (μm)
−50	10.598	200	10.053
−25	10.499	250	10.007
0	10.420	300	9.970
25	10.340	400	9.914
40	10.310	500	9.875
60	10.265	600	9.846
80	10.220	800	9.807
100	10.185	1, 000	9.783
150	10.112	1, 200	9.767

When the BRE is 0.99, the differences of the equivalent wavelengths calculated by Eq. 11 from the data in Table 2 are negligible. The propagation of the differences to BR radiance temperatures for WBRTs, t_{rad}, is less than 0.01°C in the BR temperature range from 0 to 300°C and less than 0.1°C from −50 to 1,200°C Analysis and calculation show that the equivalent wavelength is not sensitive to the BRE.

By introducing the equivalent wavelength, Eq. 10, which has the same mathematical form as Eq. 1, the formula can be rearranged to give T_{rad} directly as an analytic function of ε, T_{bb}, T_{am}, and λ_e. When the equivalent wavelength is known, the reading of a WBRT can be calculated directly from

$$T_{rad} = \frac{c_2}{\lambda_e \ln\left[\frac{1}{\frac{\varepsilon}{\exp\left(\frac{c_2}{\lambda_e T_{bb}}\right)-1} + \frac{1-\varepsilon}{\exp\left(\frac{c_2}{\lambda_e T_{am}}\right)-1}} + 1\right]} \tag{13}$$

Equation 13 should be computed using double-precision arithmetic to avoid non-negligible errors that may arise from the computation.

5 Influence of BRE on WBRT Calibration

The influence of a BRE of 0.99 on the calibrations of several typical RTs was calculated and the results are shown in Fig. 1. The legend identifies the wavelengths, spectral ranges, or pattern of spectral responsivity of the corresponding RTs.

6 Conclusion

General methods to evaluate and correct the influences of the deviation of the BRE from unity on RT calibrations and measurements were described. An approximate practical model was proposed to make corrections for a WBRT feasible. The equivalent wavelength can be introduced to simplify the mathematical model and calculation procedure for WBRTs. Mathematical expressions for the equivalent wavelength of the most popular WBRTs with a spectral range of 8 to 14 μm were given and the

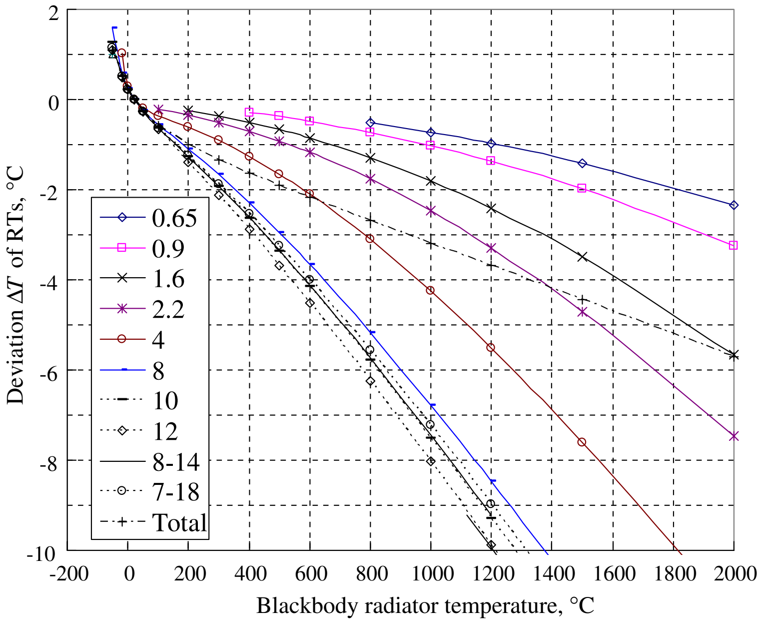

Fig. 1 Influence of BRE on the calibration of typical RTs for a BRE of 0.99 and an ambient temperature of 20°C. The legend identifies the spectral responsivity (in µm) of the corresponding RTs

equivalent wavelength was shown to be approximately 10 µm. The difference ΔT between the radiance temperature and the true temperature of a BR depends on the BRE and the RT's spectral responsivity.

For longer-wavelength RTs, the influence of a BRE of 0.99 or 0.995 on the calibration at middle or high temperatures is not negligible, may be the dominant uncertainty component for precision calibrations, and should be corrected. In the middle or high temperature range, a BR with a nominal emissivity of 0.995 is not ideal enough for a long-wavelength RT of 1% accuracy if correction for the influence of BRE deviation from unity is omitted.

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